Application of recurrence plot quantification to mineralising systems in geology

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Outline

- General motivation
- Models and experimental data
  - Benchmark models
    - *Hénon map*
    - *Gray-Scott reaction diffusion*
  - Experimental Pilot: Drill core data (minerals)
- Results
- Conclusions

Banded iron formation, Natural History Museum London
http://www.nhm.ac.uk

General Motivation

- About 40% of exploration projects are concerned in finding new profitable ore deposits
- Exploration of economically significant gold deposits becomes increasingly expensive
- Huge pressure on exploration companies

- Traditionally models vs nonlinear thinking

- Question: Can we detect stable/unstable periodic orbits using RPQA?[1]

  1. Non-stationary (spatio-temporal) data
  2. Effect of noise and …

[1] Bradley, E. and Mantilla, R., Recurrence plots and unstable periodic orbits, Chaos, 12: 596 -600, 2003

Methods used - Power spectra over recurrences; return time probability

- Wiener-Khinchin theorem[2] $$S_x(\omega) = \frac{1}{N}\left|\sum_{i=0}^{N-1} x(i)e^{-j\omega i}\right|^2 = \sum_{\tau=-\infty}^{\infty} C_x(\tau)e^{-j\omega\tau}$$
- Auto-covariance of *x(n) (with zero mean)* $$C_x(\tau) = \frac{1}{N}\sum_{i=0}^{N-1-\tau} x(i)x^*(i+\tau) = \frac{m}{N}\sum_{i=0}^{N-1-\tau} x(i)x(i+\tau)$$
- The RR =average of all recurrences f(distance matrix D) $$D(i,j) = ||x(i) - x(j)||, \text{ with } x \in \mathcal{R}^m$$
- Average distance $$d(\tau) = \frac{1}{N}\sum_i D(i, i+\tau)$$
- Relation between D and C_x $$C_x(\tau) = \sigma^2 - \frac{d^2(\tau)}{2}$$

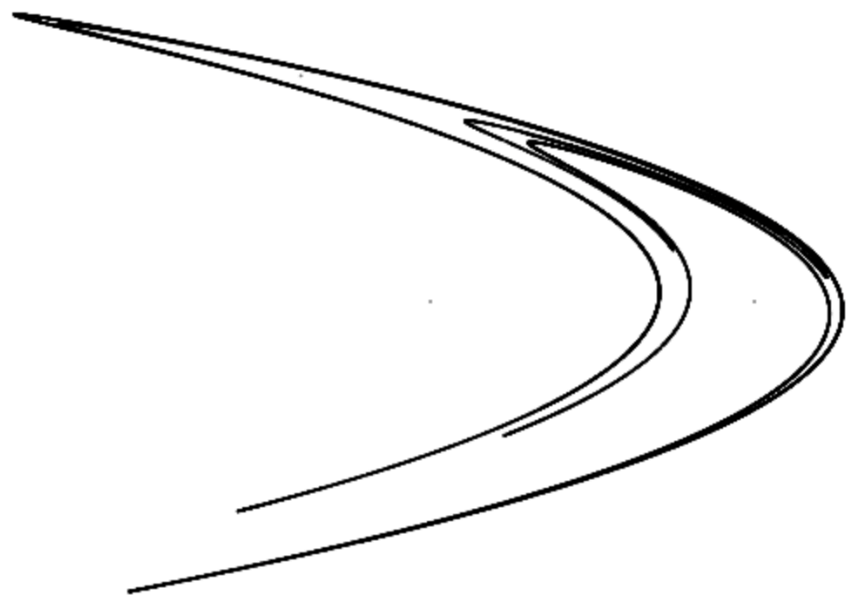
[2] Zbilut, JP, Marwan N, The Wiener-Khinchin theorem and recurrence quantification, Physics Letters, 372, 6622-6626, 2008

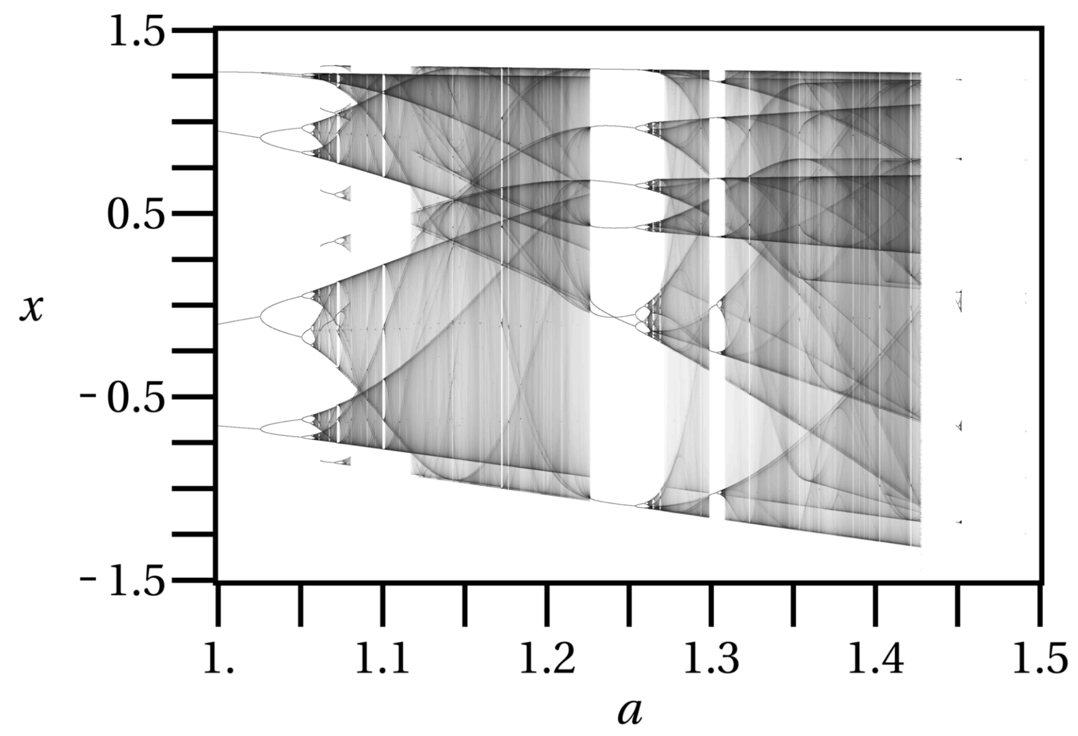
Systems analysed

- Hénon map[3]

$$(1)\ X_{i+1} = 1 - aX_i^2 + Y_i$$

$$(2)\ Y_{i+1} = bX_i$$



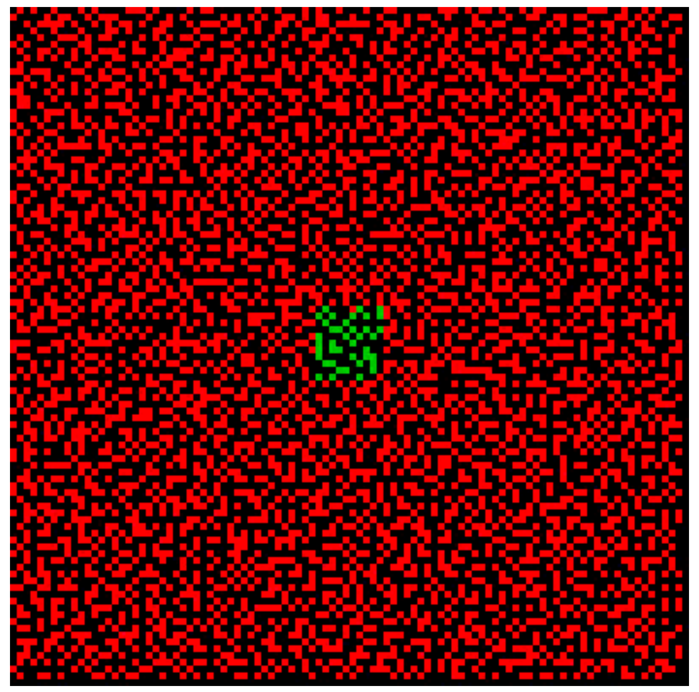


- Gray-Scott reaction-diffusion

$$U + 2V \rightarrow 3V \qquad V \rightarrow P$$

$$(1)\ \frac{\partial u}{\partial t} = r_u \nabla^2 u - uv^2 + f(1-u)$$

$$(2)\ \frac{\partial v}{\partial t} = r_v \nabla^2 v - uv^2 - (f-k)v$$



- Experimental data: Amphibole, Chlorite, Sericite

[3] **Oberst S**, Marburg S, Hoffmann N, Determining periodic orbits via nonlinear filtering and recurrence spectra in the presence of noise, EuroDyn 2017, 10 – 13 Sep, Rome, Italy.

Observation function

(1) Additive noise applied to Hénon map [3]

$$\bar{\mathbf{X}} = \mathbf{X} + p \cdot \mathbf{wn} \circ \mathbf{1}$$

(2) Multiplicative noise applied to Gray-Scott model

$$\bar{\mathbf{X}} = \mathbf{X} + p \cdot \mathbf{wn} \circ \mathbf{X}$$

[3] **Oberst S**, Marburg S, Hoffmann N, Determining periodic orbits via nonlinear filtering and recurrence spectra in the presence of noise, EuroDyn 2017, 10 – 13 Sep, Rome, Italy.

Nonlinear Filtering *ghkss* (TISEAN) [4,5]

- Locally projective noise reduction scheme: For each embedding vector exists a small correction so that $s_n - \Theta_n \in \mathcal{M}$ with a correction being orthogonal to a low dimensional manifold

- At end-pieces 'diverge' also a result of the dynamics

- Correction of centre part of delay vectors; the end pieces are left unchanged (here the influence of the negative and the positive Lyapunov exponents is the largest)

- Mostly it is sufficient to fix only the first and the last component of a delay vector (metric tensor $P_{ij} = \{0,1\}$)

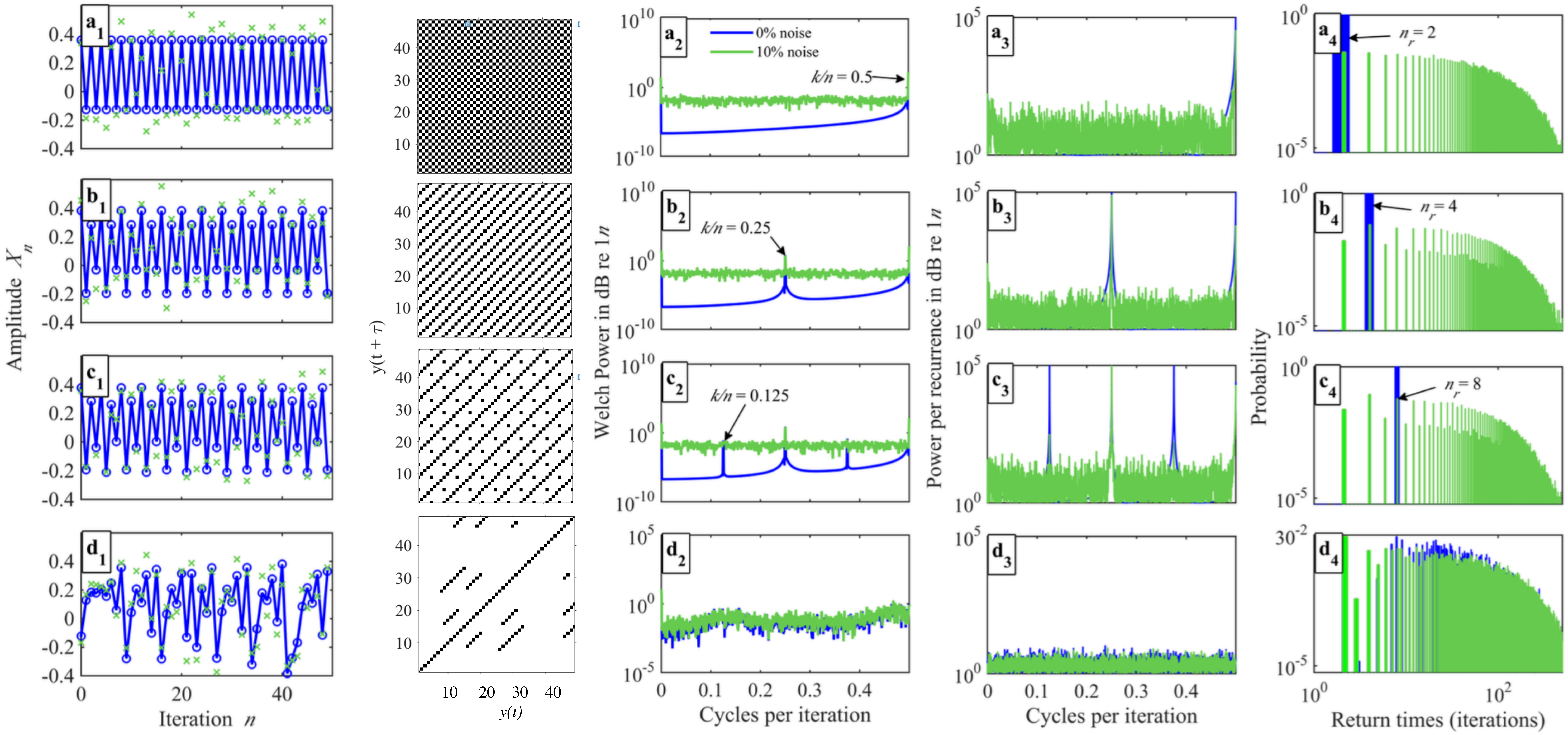
- Minimisation problem $\sum \left(\Theta_i P^{-1} \Theta_i\right) \rightarrow min$

With constraints $a_n^i (s_n - \Theta_n) + b_n^i = 0$

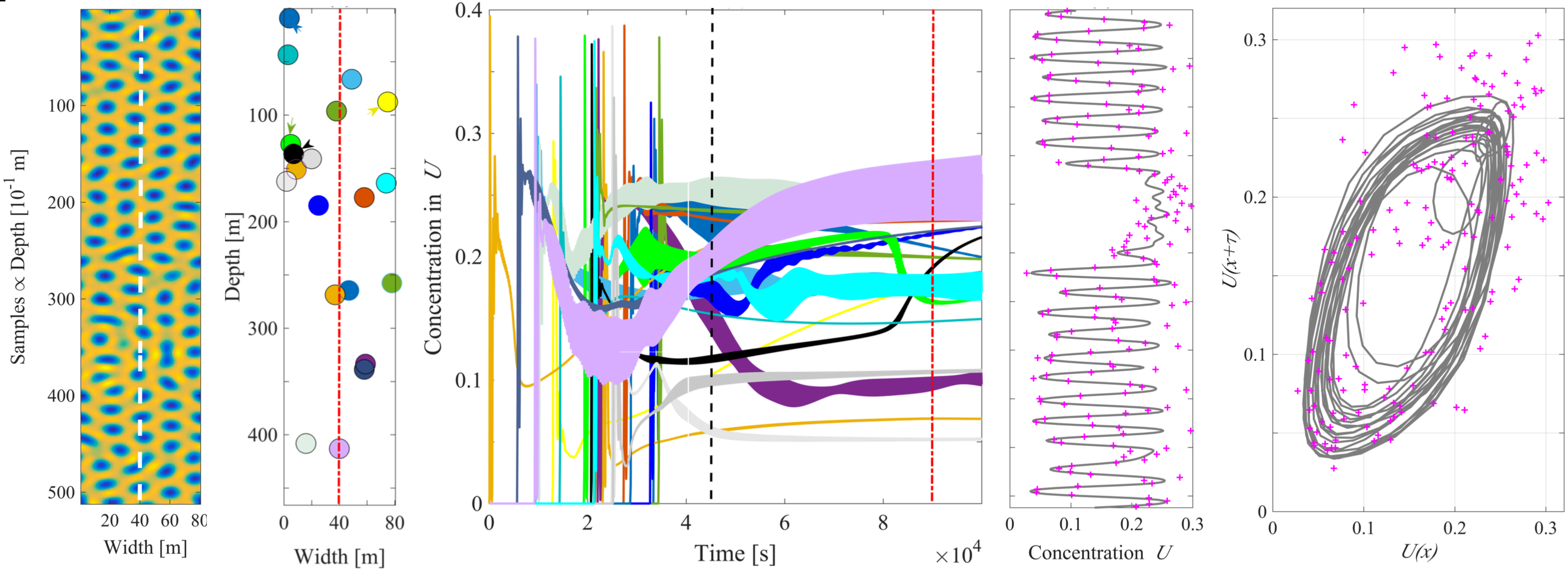
And $a_n^i P a_n^i = \delta_{ij}$

[4] Hegger R, Kantz H, Schreiber T, Practical implementation of nonlinear time series methods: The TISEAN package, CHAOS, 9:413, 1999
[5] Kantz H, Schreiber T, Hoffmann I, Buzug T, Pfister G, Nonlinear noise reduction: A case study on experimental data, Physical Review E, 48 1529 – 1538, 1993

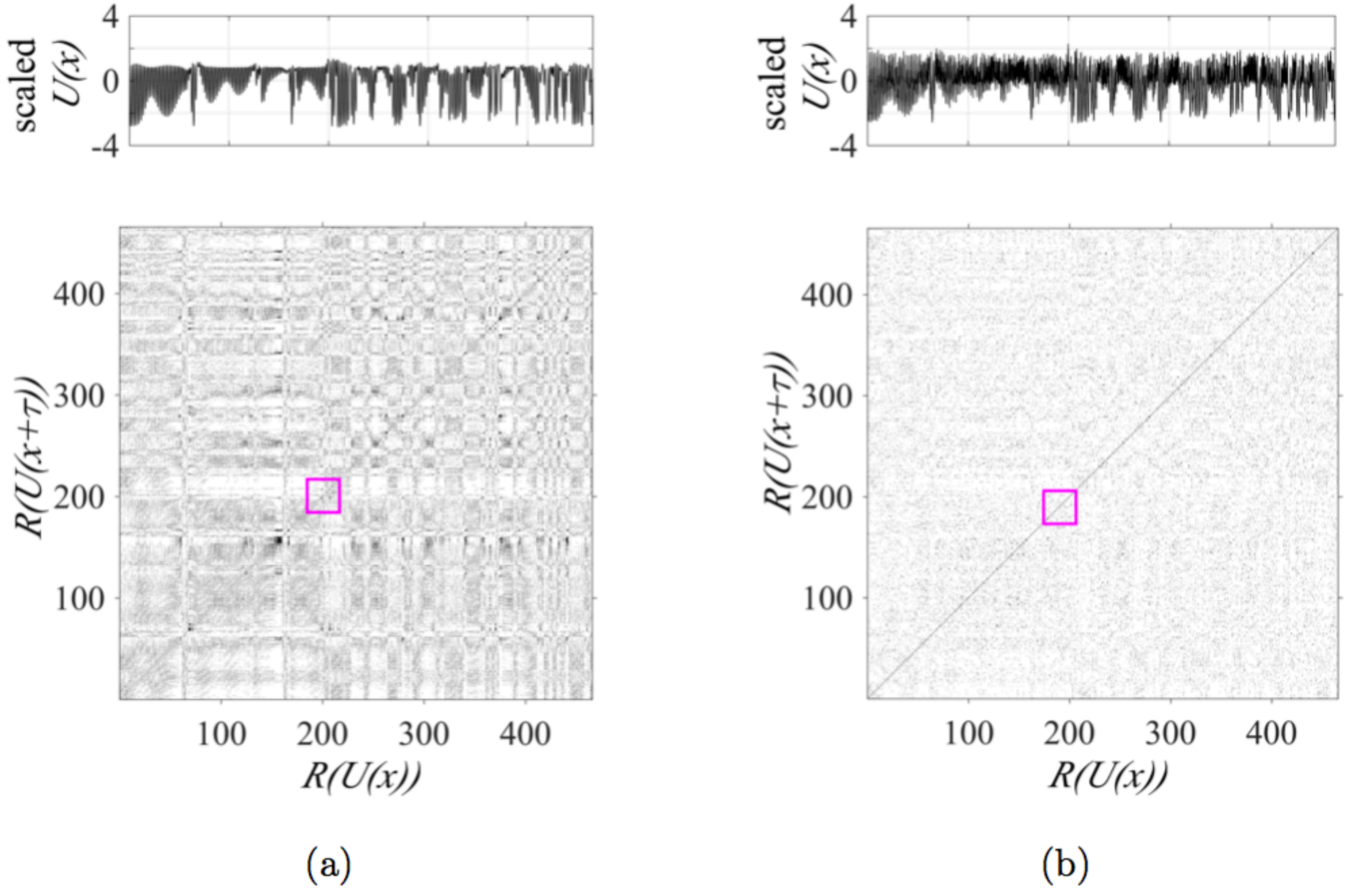
Results ‘Hénon map’



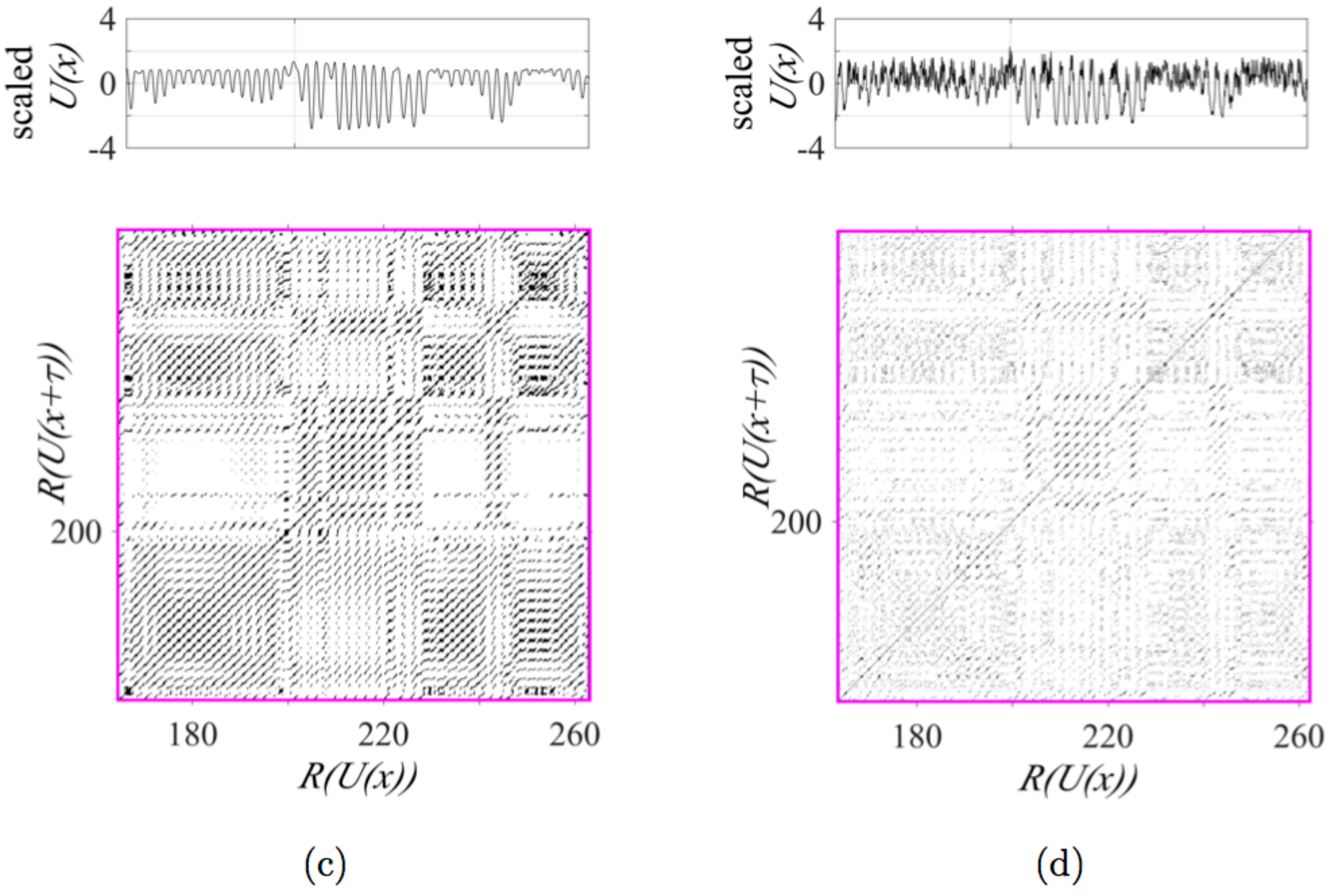
Results: ‘Gray-Scott reaction diffusion’ (1/4)



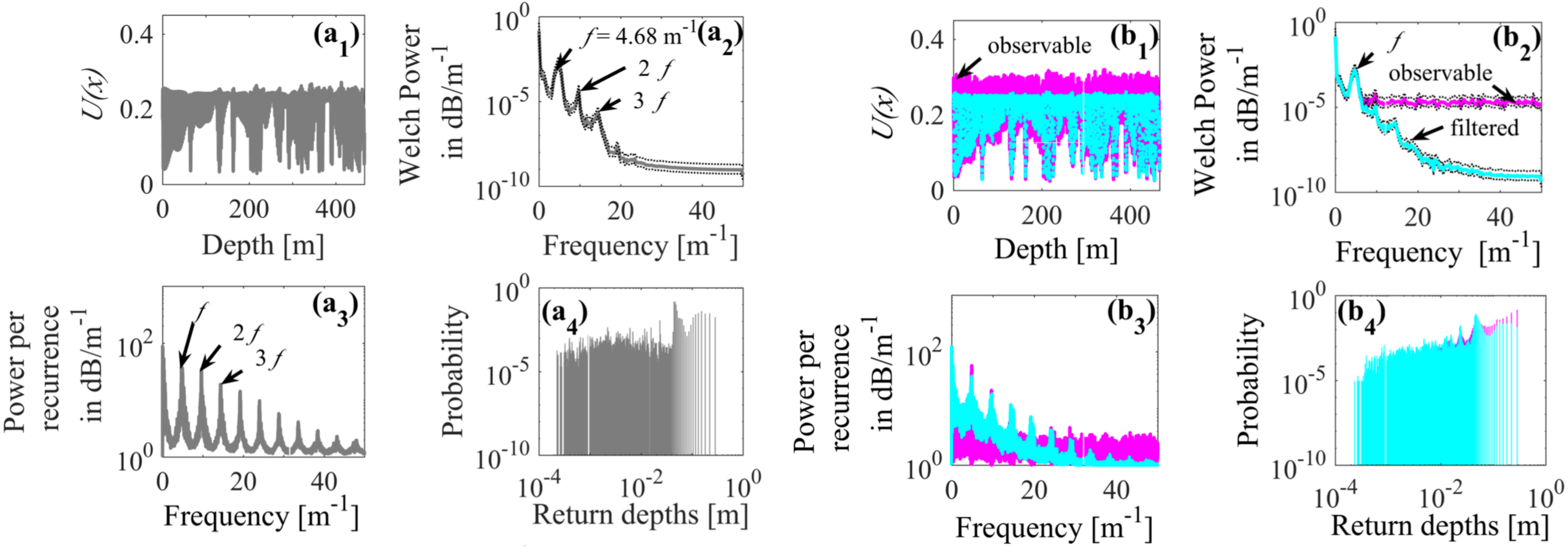
Results: ‘Gray-Scott reaction diffusion’ (2/4)



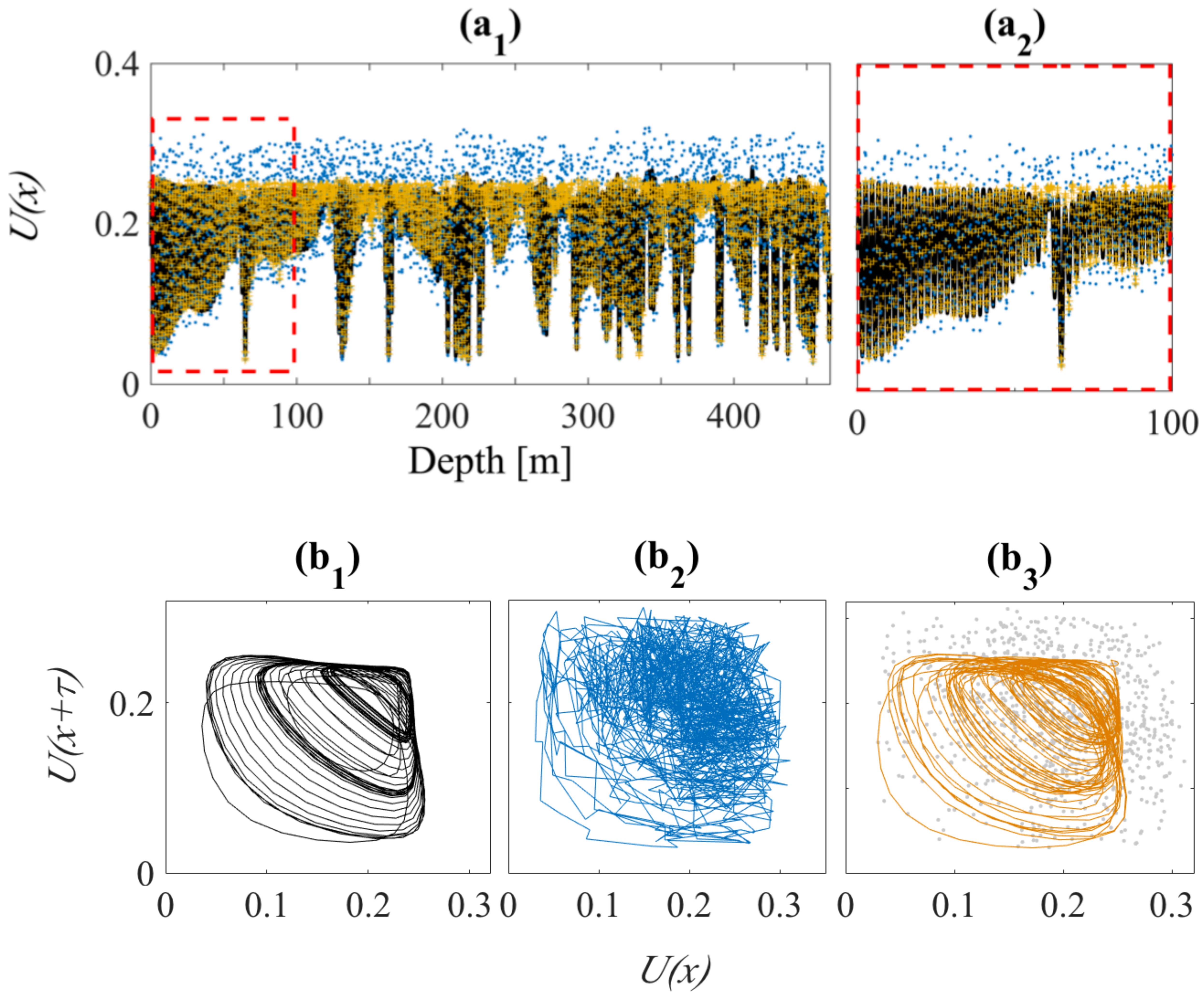
Results: ‘Gray-Scott reaction diffusion’ (2/4)



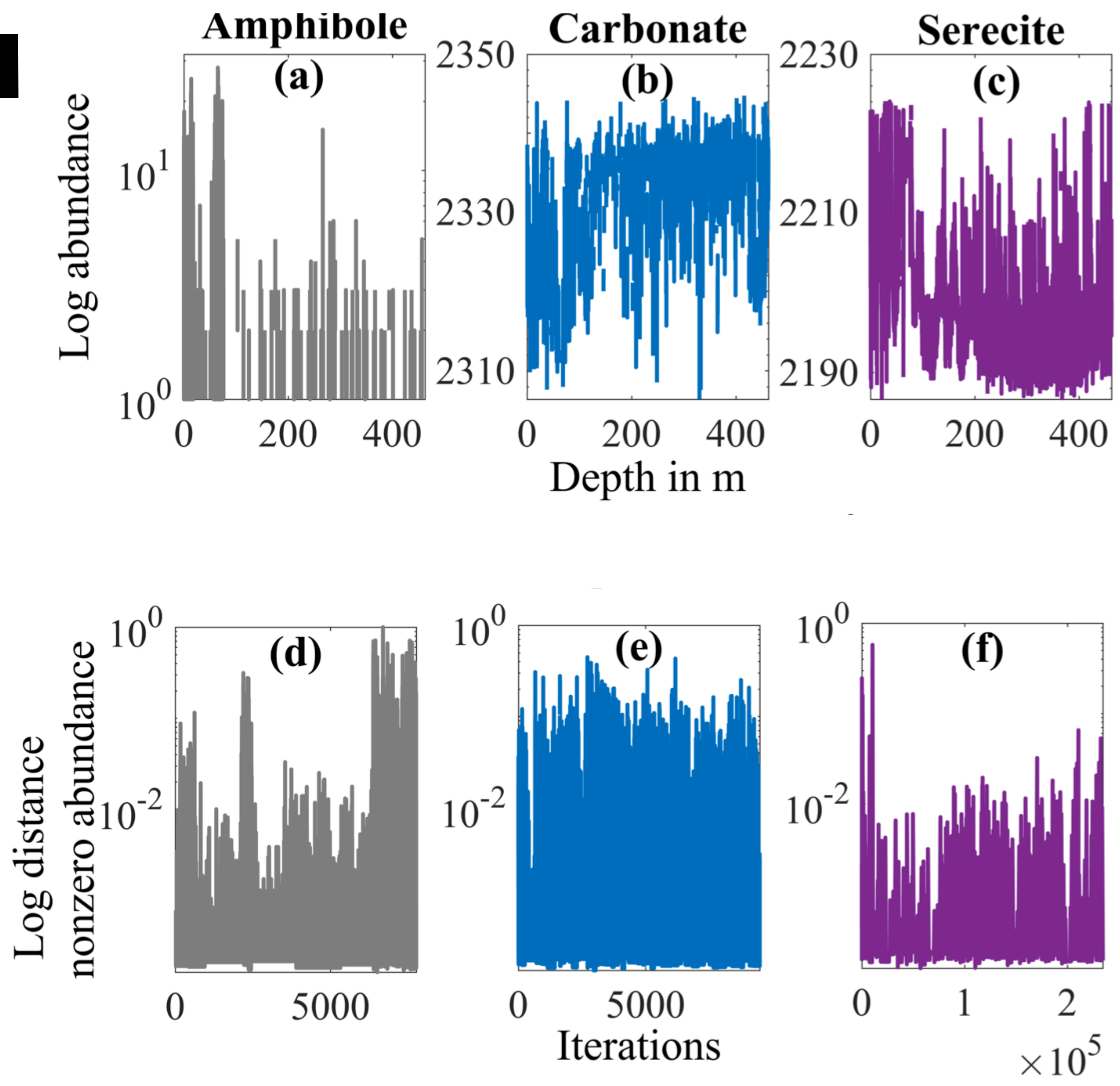
Results: ‘Gray-Scott reaction diffusion’ (3/4)

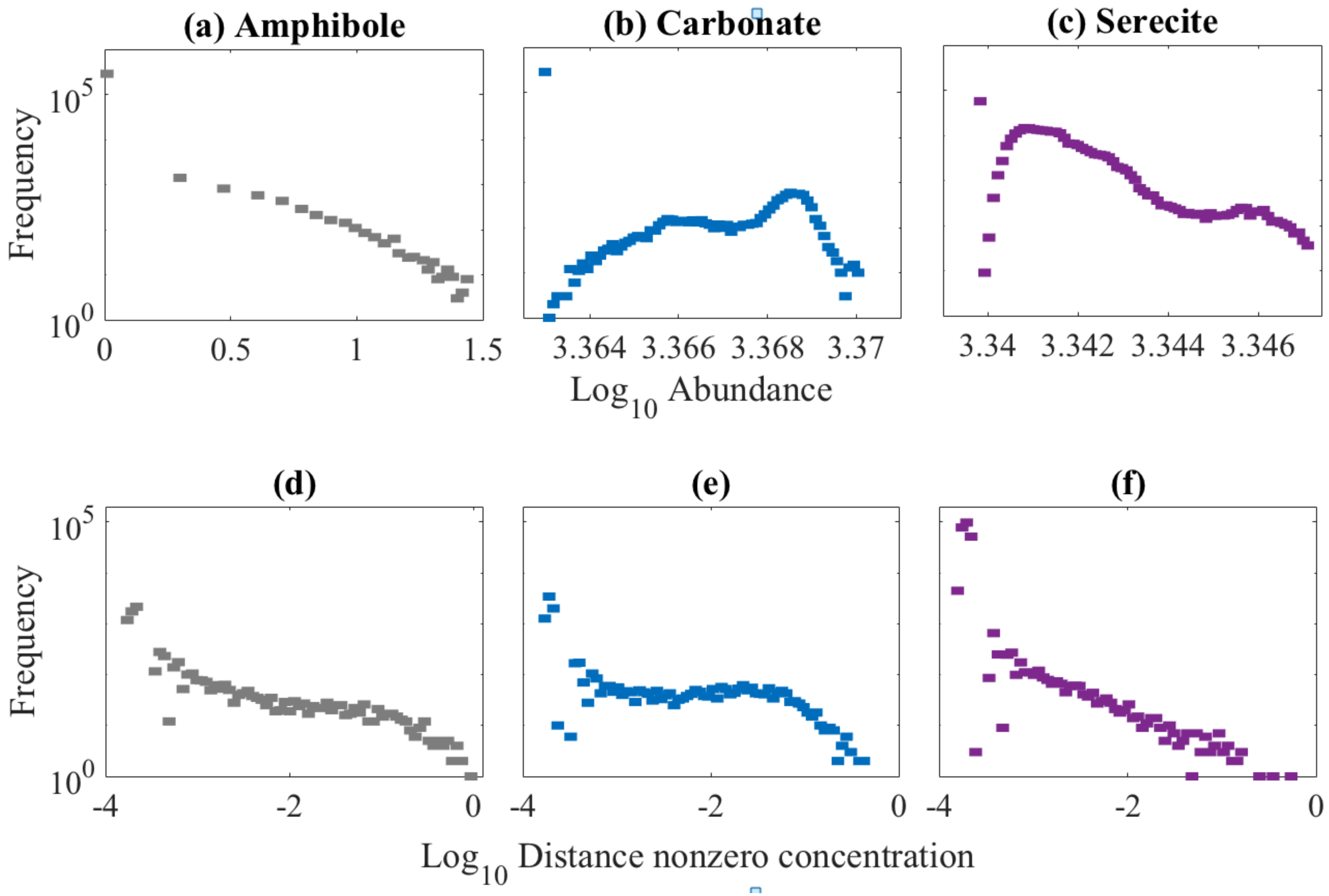


Results: ‘Gray-Scott reaction diffusion’ (2/4)

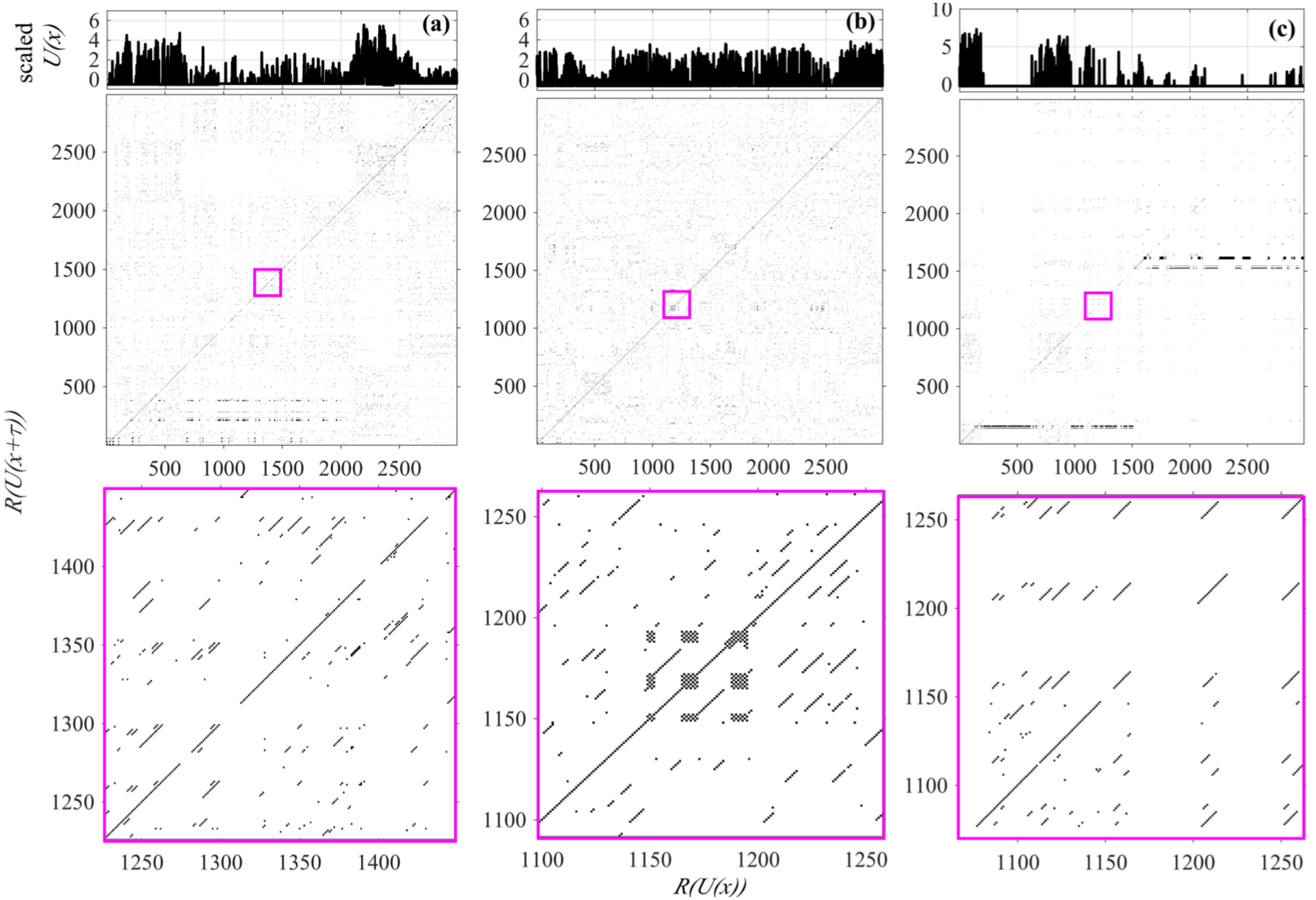


Results: ‘Drill Core Data’ (1/3)





Results ‘Drill Core Data’ (2/3)



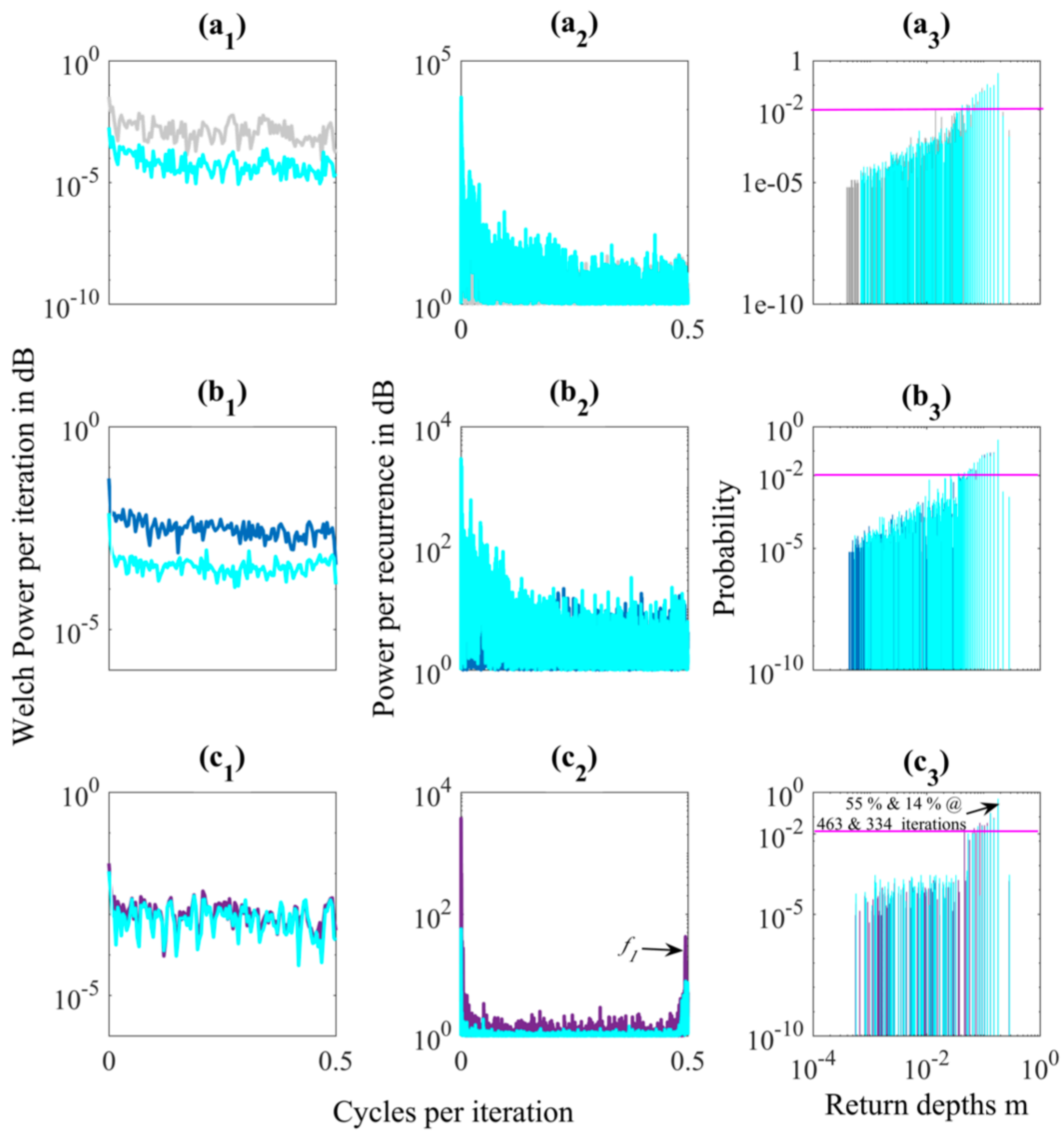
m=5, d=1, fan -norm

Results 'Drill Core Data' (3/3)

Amphibole

Carbonate

Sericite



Summary & Conclusions

- The application of recurrence plots is here conducted for the first time to mineralising data; we assumed that a mineralising system behaves similar to a *flow-driven hydrothermal chemical reactor* with dynamics far from equilibrium.

- The Hénon map as a discrete nonlinear dynamical – *stationary* - system and the Gray-Scott reaction diffusion equation as continuous spatio-temporal nonlinear dynamical– *non-stationary*- system have been used as benchmarks.

- We considered additive noise (Hénon) and multiplicative noise (Gray-Scott) to form the observer function, nonlinear filtering worked fine with those noisy systems (see also Oberst et al. 2017, EuroDyn, Rome, Italy).

- The power spectral density estimates based on recurrence measures perform better than a classical Welch spectrum: In particular for short time series the resolution is better; for non-stationary data (GSM), the RP based power spectra perform also better.

- Future work aims at taking into account a higher sampling rate and more minerals as well as chemical (digestive) data.

Thank you for your attention

Fordite, Michigan (US) 😆
http://www.geologypage.com